χ^2 Distribution

What is χ^2 **Distribution**?

The χ^2 **distribution** is defined by the sum of the squared of randomly selected variables Z from the standard normal probability distribution. When you sample from k independent standard normal probability distributions, we get

$$\chi^2_k = Z_1^2 + Z_2^2 + \cdots + Z_k^2 = \sum_{i=1}^{k} Z_i^2$$

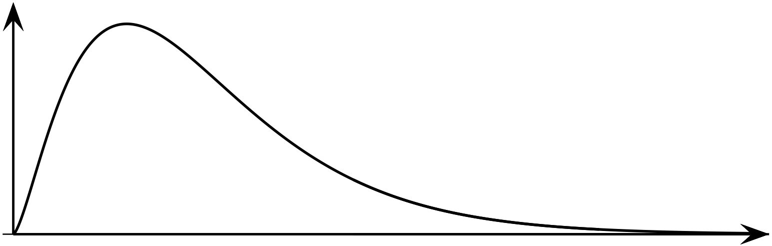
where k is the degrees of freedom

The χ^2 **distribution** is a method to find the probability when we are comparing observed data to what it is expected under certain assumption.

What does χ^2 **Distribution** density curve look like?

- The density curve is not symmetric.
- The density curve is not bell-shaped.
- The total area under the curve is 1.
- It comes with degrees of freedom.
- The density curve may look bell-shaped as degrees of freedom increases.
- The density curve begins at 0 and it is skewed to the right when degrees of freedom is greater than 2.

Here is how the χ^2 distribution curve look like with degrees of freedom $k > 2$.



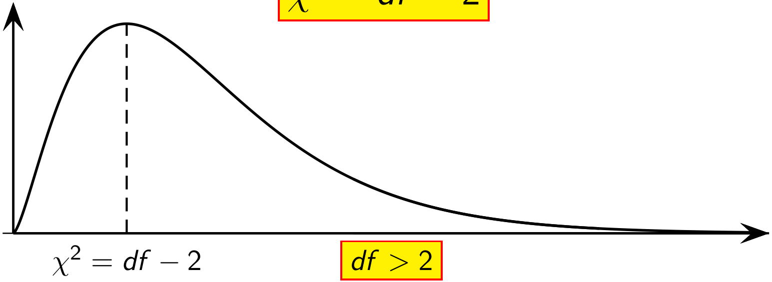
The χ^2 distribution has a mean $\mu = k$.

The χ^2 distribution has a variance $\sigma^2 = 2k$.

Where does χ^2 **Distribution** curve peak?

- When *df* $= 1$, the χ^2 distribution curve $\to \infty$ as $\chi^2 \to 0$.
- When *df* $= 2$, the χ^2 distribution curve $\to 0.5$ as $\chi^2 \to 0$.
- When *df* > 2, the χ^2 distribution curve has a peak point at

$$\chi^2 = df - 2$$



Example:

Consider a χ^2 distribution with degrees of freedom $k = 8$.

- Find χ^2 of the peak point of its density curve.
- Find the mean of the distribution.
- Find the standard deviation of the distribution.

Solution:

With degrees of freedom $k = 8$,

- The peak point happens at $\chi^2 = k - 2 = 8 - 2 = 6$.
- The mean of the distribution is $\mu = k = 8$.
- The standard deviation of the distribution is $\sigma = \sqrt{\sigma^2} = \sqrt{2k} = \sqrt{2 \cdot 8} = \sqrt{16} = 4$.

Probability with χ^2 Distribution using TI:

Press **2ND**, then VARS and $\downarrow$, to select χ^2cdf.

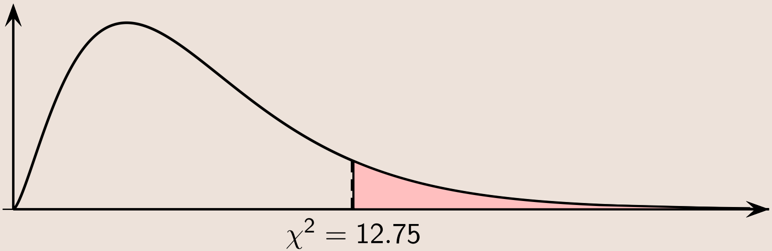
Probability Type	TI Command
$P(\chi^2 < a)$	$\chi^2\text{cdf}(0, a, df)$
$P(\chi^2 > b)$	$\chi^2\text{cdf}(b, E99, df)$
$P(a < \chi^2 < b)$	$\chi^2\text{cdf}(a, b, df)$
$P(\chi^2 < a \text{ or } \chi^2 > b)$	$1 - \chi^2\text{cdf}(a, b, df)$

Example:

Find $P(\chi^2 > 12.75)$ with $df = 7$. Round to 3-decimal places.

Solution:

We start by drawing the χ^2 distribution curve, then label and shade accordingly.



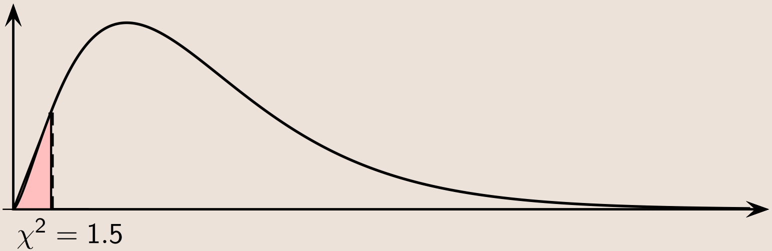
Now we can use the TI command,
$P(\chi^2 > 12.75) = \chi^2 cdf(12.75, E99, 7) \approx 0.078$.

Example:

Find $P(\chi^2 < 1.5)$ with $df = 6$. Round to 3-decimal places.

Solution:

We start by drawing the χ^2 distribution curve, then label and shade accordingly.



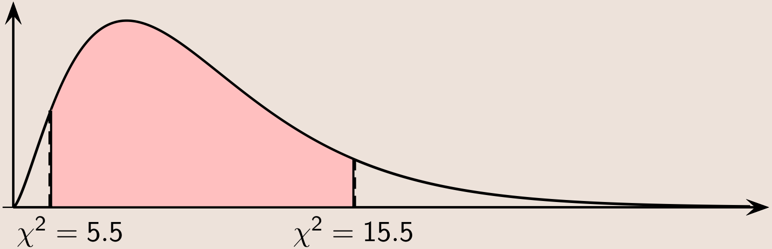
Now we can use the TI command,
$P(\chi^2 < 1.5) = \chi^2 cdf(0, 1.5, 6) \approx 0.041$.

Example:

Find $P(5.5 < \chi^2 < 15.5)$, with $df = 9$. Round to 3-decimal places.

Solution:

We start by drawing the χ^2 distribution curve, then label and shade accordingly.



Now we can use the TI command,

$P(5.5 < \chi^2 < 15.5) = \chi^2 cdf(5.5, 15.5, 9) \approx 0.711$.